INFLUENCE OF RANDOM DEVIATIONS FROM THE OPTIMAL THRUST PROGRAM ON THE MOTION IN A GRAVITATIONAL FIELD OF A VARIABLE-MASS BODY WITH CONSTANT POWER CONSUMPTION

(VLIIANIE SLUCHAINYKH OTKLONENII OT OPTIMAL'NOI PROGRAMMY PO TIAGE NA DVIZHENIE TELA PEREMENNOI MASSY S POSTOIANNOI ZATRATOI MOSHCHNOSTI V GRAVITATSIONNOM POLE)

PMM Vol.27, No.1, 1963, pp. 27-32

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(Received July 6, 1962)

In [1] the influence of random power reductions on the optimal parameters of the motion of a variable-mass body in a gravitational field, was studied. Below we consider the problem of the determination of the minimum averaged value of the increase of a characteristic functional which is produced by random errors in the realization of the extremal law of variation of the reactive thrust during the motion of a variable-mass body with constant power consumption.

1. Optimal unperturbed motion. Let the power of the reactive jet be constant

$$N = \frac{1}{2} qV^2 \equiv \text{const} \tag{1.1}$$

Here q is the fuel consumption per unit time, V is the fuel discharge velocity.

In this case the problem of determining the minimum total relative weight ($G^\circ = G_N^\circ + G_M^\circ$) of the power source ($G_N^\circ = \alpha N/G_0$, where α is the specific gravity of the power source, independent of N, and G_0 is the weight of the body at the initial instant $t = 0$), and of determining the required fuel supply ($G_M^\circ = 1 - (G_k/G_0)$, where G_k is the weight of the body at the end of the motion, when $t = T$), reduces to the following two problems [2,3].

1. To find such a law for the time variation of the reactive thrust acceleration $\mathbf{a}_*^{(0)}(t)$ (the thrust is divided by the current mass), and such a trajectory $\mathbf{r}_*^{(0)}(t)$ that when transferring between two fixed points $\{\mathbf{r}_0, \dot{\mathbf{r}}_0\}$ and $\{\mathbf{r}_k, \dot{\mathbf{r}}_k\}$ in the phase space within a given time T, the value of the functional

$$\Phi = \frac{\alpha}{2g}\int_0^T a^2 dt \qquad (1.2)$$

(where g is the gravitational acceleration at the Earth's surface), has a minimum

$$\Phi\,[\mathbf{a}_*^{(0)}(t)] = \Phi_{min}.$$

2. For a known value of functional Φ to choose such values of the relative weight of the power source G_{N*}° and of the required fuel supply G_{M*}°, that their sum is minimal.

In general, the solution of problem 1 is determined numerically, but in the case of motion in a uniform gravitational field ($\ddot{\mathbf{r}} = \mathbf{a} - \mathbf{g}_0$, where $\mathbf{g}_0$ is the constant acceleration vector of the gravitational force) the solution is expressed by the formulas

$$\begin{aligned} \mathbf{a}_*^{(0)}(t) &= \mathbf{g}_0 + 6\left(\frac{\mathbf{r}_k - \mathbf{r}_0}{T^2} - \frac{1}{3}\frac{\dot{\mathbf{r}}_k + 2\dot{\mathbf{r}}_0}{T}\right) - 6\left(2\frac{\mathbf{r}_k - \mathbf{r}_0}{T^2} - \frac{\dot{\mathbf{r}}_k + \dot{\mathbf{r}}_0}{T}\right)\frac{t}{T} \\ \mathbf{r}_*^{(0)}(t) &= \mathbf{r}_0 + \dot{\mathbf{r}}_0 t + 3\left(\frac{\mathbf{r}_k - \mathbf{r}_0}{T^2} - \frac{1}{3}\frac{\dot{\mathbf{r}}_k + 2\dot{\mathbf{r}}_0}{T}\right)t^2 - \left(2\frac{\mathbf{r}_k - \mathbf{r}_0}{T^2} - \frac{\dot{\mathbf{r}}_k + \dot{\mathbf{r}}_0}{T}\right)\frac{t^3}{T} \qquad (1.3) \\ \Phi_{\min} &= \frac{6\alpha}{gT^3}\Big\{(\mathbf{r}_k - \mathbf{r}_0)^2 - (\mathbf{r}_k - \mathbf{r}_0, \dot{\mathbf{r}}_k + \dot{\mathbf{r}}_0)\,T + [\dot{r}_k^2 + (\dot{\mathbf{r}}_k, \dot{\mathbf{r}}_0) + \dot{\mathbf{r}}_0^2]\frac{T^2}{3} + \\ &\quad + (\mathbf{g}_0, \dot{\mathbf{r}}_k - \dot{\mathbf{r}}_0)\frac{T^3}{6} + g_0^2\frac{T^4}{12}\Big\} \end{aligned}$$

Problem 2 has a simple analytic solution

$$G_{N*}^{\circ} = \sqrt{\Phi} - \Phi, \qquad G_{M*}^{\circ} = \sqrt{\Phi} \qquad (0 < \Phi < 1) \qquad (1.4)$$

As an example of the original unperturbed motion, let us consider uniform transfer in a force-free field ($\mathbf{g}_0 = 0$) over a given distance L within a fixed time T, with zero initial and final velocities. For such a motion formulas (1.3) take the form

$$a_*^{(0)}(t) = \frac{6L}{T^2}\left(1 - 2\frac{t}{T}\right), \qquad r_*^{(0)}(t) = L\frac{t^2}{T^2}\left(3 - 2\frac{t}{T}\right), \qquad \Phi_{\min} = \frac{6\alpha L^2}{gT^3} \qquad (1.5)$$

2. Deviation from the computed trajectory. The extremal law of variation of the thrust acceleration $\mathbf{a}_*^{(0)}(t)$, found as a result of solving problem 1, will be realized with several errors $\delta\mathbf{a}(t)$ which lead to a deviation of the actual trajectory $\mathbf{r}(t)$ from the computed one

$$\delta_a \dot{\mathbf{r}}(t) = \dot{\mathbf{r}}(t) - \dot{\mathbf{r}}_*^{(0)}(t), \qquad \delta_a \mathbf{r}(t) = \mathbf{r}(t) - \mathbf{r}_*^{(0)}(t) \tag{2.1}$$

In the case of a uniform gravitational field, by considering the error $\delta\mathbf{a}$ to be a random vector with uncorrelated components, we can investigate the motion of each coordinate independently.

Let $\delta a(t)$ be a random function such that

$$M[\delta a(t)] \equiv 0, \qquad K[\delta a(t), \delta a(t')] = \sigma_a^2 A(t) A(t') \exp(-|t-t'|/\delta t) \tag{2.2}$$

where M and K are the symbols for the mathematical expectation and for the correlation function; σ_a and δt are constants which characterize the accuracy and the speed of the control system which ensures the fulfillment of time-variation program for the thrust acceleration; $A(t)$ is a nonrandom function determined by the type of control: $A(t) = a(t)$ for relative error control, $A(t) \equiv$ const for absolute error control (later we shall study absolute error control and assume that $A(t) \equiv L/T^2$).

The dispersion from the computed trajectory of the random errors (2.1) produced by errors δa of such a form, are determined by the formulas

$$D[\delta_a v(\theta)] = 2\sigma_a^2 \tau [\theta - \tau(1 - e^{-\theta/\tau})] \qquad \left(v = \frac{rT}{L},\ \theta = \frac{t}{T},\ \tau = \frac{\delta t}{T}\right) \tag{2.3}$$

$$D[\delta_a s(\theta)] = \frac{2}{3}\sigma_a^2 \tau \left\{\theta^2 - 3\tau\left[\frac{1}{2}\theta^2 + \tau\theta e^{-\theta/\tau} - \tau^2(1 - e^{-\theta/\tau})\right]\right\} \quad \left(s = \frac{r}{L}\right)$$

The deviations from the computed trajectory are caused not only by the errors $\delta\mathbf{a}$ but also by the inaccurate realization of the initial conditions

$$\delta_0 \dot{\mathbf{r}} = \dot{\mathbf{r}}(0) - \dot{\mathbf{r}}_0, \qquad \delta_0 \mathbf{r} = \mathbf{r}(0) - \mathbf{r}_0 \tag{2.4}$$

The errors $\delta_0 \dot{\mathbf{r}}$ and $\delta_0 \mathbf{r}$ are assumed to be independent random vectors whose components are mutually uncorrelated and are distributed around zero with known mean square deviations; in order to study uniform motion, it is assumed that

$$M(\delta_0 v) = M(\delta_0 s) = 0, \qquad D(\delta_0 v) = \sigma_{0v}^2, \qquad D(\delta_0 s) = \sigma_{0s}^2 \tag{2.5}$$

If at the end of the motion it is expected that the deviations of the real trajectory from the computed one, will be greater than the given admissible deviations $\delta\dot{r}_{max}$ and δr_{max}, then, it will be necessary to correct the trajectory during the motion. In the example being considered, this is tantamount to the violation of the system of inequalities

$$2\sigma_a^2\tau + \sigma_{0v}^2 \leqslant \delta v_{\max}^2, \qquad \frac{2}{3}\sigma_a^2\,\tau + \sigma_{0v}^2 + \sigma_{0s}^2 \leqslant \delta s_{\max} \tag{2.6}$$

which is obtained from formulas (2.3) and (2.5), and moreover, where terms of order τ^2 or higher have been neglected since $\delta t \ll T$.

3. Measurement of the position and the velocity of the body. In order to effect the correction to the trajectory it is necessary to know the actual values of the coordinates and of the velocity of the body. Individual measurements of these quantities are considered to be of unsatisfactory accuracy, and, therefore, it is proposed that they be made regularly and sufficiently often, and that the results of all the measurements made between the instants t_{i-1} and t_i ($i = 1, 2, \ldots, \mu$) be enumerated at the instant t_i and averaged. Nevertheless, certain mistakes will still remain

$$\delta_n\dot{\mathbf{r}}_i = \dot{\boldsymbol{\rho}}\,(t_i) - \dot{\mathbf{r}}\,(t_i), \qquad \delta_n\mathbf{r}_i = \boldsymbol{\rho}\,(t_i) - \mathbf{r}\,(t_i) \tag{3.1}$$

where $\boldsymbol{\rho}(t_i)$ and $\dot{\boldsymbol{\rho}}(t_i)$ are the radius and the velocity obtained as a result of averaging the measurements. In the system being studied, it is considered that

$$M\,(\delta_n v_i) = M\,(\delta_n s_i) = 0, \qquad D\,(\delta_n v_i) = (\sigma_{nv}^{(i)})^2, \qquad D\,(\delta_n s_i) = (\sigma_{ns}^{(i)})^2 \tag{3.2}$$

and that the errors $\delta_n v_i$, $\delta_n s_i$ ($i = 1, \ldots, \mu$) are mutually uncorrelated.

4. Optimal corrections to the reactive thrust acceleration program. As a result of averaging the measurements at the instant t_i ($i = 1, \ldots, \mu$), we know, with some degree of accuracy, the deviations of the actual trajectory from the $(i-1)$st computed trajectory $\mathbf{r}_*^{(i-1)}(t)$

$$\Delta\dot{\mathbf{r}}_i = \dot{\boldsymbol{\rho}}\,(t_i) - \dot{\mathbf{r}}_*^{(i-1)}\,(t_i), \qquad \Delta\mathbf{r}_i = \boldsymbol{\rho}\,(t_i) - \mathbf{r}_*^{(i-1)}\,(t_i) \tag{4.1}$$

From the optimality condition "in the large": the correction $\Delta\mathbf{a}_*^{(i)}(t) = \mathbf{a}_*^{(i)}(t) - \mathbf{a}_*^{(i-1)}(t)$, computed at the instant t_i (the ith correction instant), must ensure the minimum of integral (1.2) in the interval (t_i, T) when transferring between the points $\{\boldsymbol{\rho}(t_i), \dot{\boldsymbol{\rho}}(t_i)\}$ and $\{\mathbf{r}_k, \dot{\mathbf{r}}_k\}$ in the phase space. We assume that no errors are introduced when computing $\Delta\mathbf{a}_*^{(i)}$. For uniform motion in a force-free field

$$\Delta a_*^{(i)}\,(\theta) = \frac{6L}{T^2}\frac{1}{1-\theta_i}\left[\left(2\frac{\Delta s_i}{1-\theta_i} + \Delta v_i\right)\frac{\theta-\theta_i}{1-\theta_i} - \left(\frac{\Delta s_i}{1-\theta_i} + \frac{2}{3}\,\Delta v_i\right)\right] \tag{4.2}$$

The program $\mathbf{a}_*^{(i)}(t)$, which at the instant t_{i+1} is replaced by a new optimal program $\mathbf{a}_*^{(i+1)}(t)$, should be effected in the time interval between t_i and t_{i+1}, etc.

5. Increase of the functional. Optimal distribution of the correction instants. Expression (1.2) for functional Φ, in the case of motion with errors and corrections, can be represented in the form

$$\Phi = \frac{\alpha}{2g}\sum_{i=0}^{\mu}\int_{t_i}^{t_{i+1}} (\mathbf{a}_*^{(i)} + \delta\mathbf{a})^2\, dt \qquad (t_0 = 0,\ t_{\mu+1} = T) \tag{5.1}$$

In a force-free field the equations for $\Delta a_*^{(i)}$ are linear, and therefore

$$a_*^{(i)}(t) = a_*^{(0)}(t) + \Delta a_*^{(1)}(t) + \ldots + \Delta a_*^{(i)}(t) \tag{5.2}$$

This allows us to transform the functional Φ in the following way:

$$\begin{aligned}\Delta\Phi = \Phi - \Phi_{\min} = \frac{\alpha}{2g}\Big\{2\Big[\sum_{i=1}^{\mu}\int_{t_i}^{T} a_*^{(0)}\Delta a_*^{(i)}\,dt + \int_0^T a_*^{(0)}\delta a\,dt\Big] +\\ +2\Big[\sum_{i=3}^{\mu}\int_{t_i}^{T}\Delta a_*^{(i)}\Big(\sum_{j=1}^{i-2}\Delta a_*^{(j)}\Big)dt + \sum_{i=1}^{\mu}\int_{t_i}^{T}\Delta a_*^{(i)}\delta a\,dt\Big] +\\ +\Big[2\sum_{i=2}^{\mu}\int_{t_i}^{T}\Delta a_*^{(i)}\Delta a_*^{(i-1)}\,dt + \sum_{i=1}^{\mu}\int_{t_i}^{T}(\Delta a_*^{(i)})^2\,dt + \int_0^T(\delta a)^2\,dt\Big]\Big\}\end{aligned} \tag{5.3}$$

By supposing that $\Delta\Phi/\Phi_{\min} \ll 1$, and by restricting ourselves to terms which are linear in $\Delta\Phi/\Phi_{\min}$, with the help of (1.4) we get an approximate expression for the minimal weight $G_{\min}^{\circ}$ for a given Φ

$$G_{\min}^{\circ} = G_{N*}^{\circ} + G_{M*}^{\circ} \approx 2\sqrt{\Phi_{\min}} - \Phi_{\min} + \Delta\Phi\left(\frac{1}{\sqrt{\Phi_{\min}}} - 1\right) \tag{5.4}$$

In order to minimize the average with respect to $\Delta\Phi$ of the quantity $G_{\min}^{\circ}$, it is necessary to define a distribution of the correction instants t_i ($i = 1, \ldots, \mu$) and their number μ_{opt}, such that under the condition which ensures a given accuracy at the end of the motion (on the average), the mathematical expectation of $\Delta\Phi$ is minimal. In a force-free field the deviations measured at the instant t_i, Δv_i, Δs_i, of the real trajectory from the computed one, from which the correction $\Delta a_*^{(i)}$ is calculated (see formula (4.2)), are composed of the deviations produced by errors $\delta a(t)$ and of the errors at the $(i-1)$st and ith measurements

$$\Delta v_i = \frac{T}{L}\int\limits_{t_{i-1}}^{t_i} \delta a dt + \delta_n v_i - \delta_n v_{i-1}$$
$$\Delta s_i = \frac{1}{L}\int\limits_{t_{i-1}}^{t_i} dt \int\limits_{t_{i-1}}^{t} \delta a dt + \delta_n s_i - \delta_n s_{i-1} - \delta_n v_{i-1}(\theta_i - \theta_{i-1}) \quad (5.5)$$

The mathematical expectation of the terms contained in the first square brackets of formula (5.3), equals zero since the integrands in these terms are linear in the random quantities Δv_i, Δs_i and δa, whose mathematical expectations equal zero. By considering that the random deviations $\delta a(t)$ on both sides of the instant t_i at which we transfer to a new program $a_*^{(i)}(t)$, are mutually uncorrelated

$$K[\delta a(t), \delta a(t')] = 0 \quad \text{for } t_{i-1} < t < t_i,\ t_{j-1} < t' < t_j,\ i \neq j \quad (5.6)$$

then, we get the same result also for the mathematical expectation of the second square brackets of formula (5.3), since the integrands in these terms consist of products of independent random variables, whose mathematical expectations equal zero.

The first two terms in the last square brackets of formula (5.3) are expressed in terms of the deviations Δv_{i-1}, Δv_i and Δs_{i-1} Δs_i, thus

$$\int\limits_{t_i}^{T} \Delta a_*^{(i)} \Delta a_*^{(i-1)} dt = \frac{12L^2}{T^3}\frac{1}{1-\theta_{i-1}}\left[\frac{\Delta s_i \Delta s_{i-1}}{(1-\theta_{i-1})^2} + \frac{\Delta s_i \Delta v_{i-1}}{2(1-\theta_{i-1})} + \right.$$
$$\left. + \left(1 - 2\frac{\theta_i - \theta_{i-1}}{1-\theta_{i-1}}\right)\frac{\Delta v_i \Delta s_{i-1}}{2(1-\theta_{i-1})} + \left(\frac{1}{3} - \frac{1}{2}\frac{\theta_i - \theta_{i-1}}{1-\theta_{i-1}}\right)\Delta v_i \Delta v_{i-1}\right] \quad (5.7)$$
$$\int\limits_{t_i}^{T} (\Delta a_*^{(i)})^2 dt = \frac{12L^2}{T^3}\frac{1}{1-\theta_i}\left[\frac{(\Delta s_i)^2}{(1-\theta_i)^2} + \frac{\Delta s_i \Delta v_i}{1-\theta_i} + \frac{1}{3}(\Delta v_i)^2\right]$$

Therefore, the mathematical expectation of the increase in the functional, with an accuracy up to quadratic terms in τ, equals

$$M(\Delta\Phi) = \frac{6\alpha L^2}{gT^3}\left\{\sum_{i=1}^{\mu}\left[\frac{\sigma_n^2\tau}{3}(2\xi_i^3 - 3\xi_i^2 + 2\xi_i - 1) + \right.\right.$$
$$\left. + \frac{(\sigma_{nv}^{(i-1)})^2}{1-\theta_i}\xi_i(\xi_i - 1) + \frac{(\sigma_{nv}^{(i-1)})^2 - (\sigma_{nv}^{(i)})^2}{3(1-\theta_i)} + \frac{(\sigma_{ns}^{(i-1)})^2 - (\sigma_{ns}^{(i)})^2}{(1-\theta_i)^3}\right] +$$
$$\left. + \frac{\sigma_n^2}{12} + \frac{2}{3}\frac{(\sigma_{nv}^{(\mu)})^2}{1-\theta_\mu} + 2\frac{(\sigma_{ns}^{(\mu)})^2}{(1-\theta_\mu)^3}\right\} \quad \left(\xi_i = \frac{1-\theta_{i-1}}{1-\theta_i},\ \sigma_{nv}^{(0)} = \sigma_{0v},\ \sigma_{ns}^{(0)} = \sigma_{0s}\right) \quad (5.8)$$

When $t = T$ ($\theta = 1$) the trajectory must hit on the given region of finite values of the coordinates and of the velocity, i.e. the expected deviations from the computed trajectory at the end of the motion should be bounded from above; this condition can be written in the form of a system of two inequalities

$$2\sigma_a^2\tau(1-\theta_\mu) + (\sigma_{nv}^{(\mu)})^2 \leqslant (\delta v_{\max})^2 \qquad (5.9)$$
$$\frac{2}{3}\sigma_a^2\tau(1-\theta_\mu)^3 + (\sigma_{nv}^{(\mu)})^2(1-\theta_\mu)^2 + (\sigma_{ns}^{(\mu)})^2 \leqslant \delta s_{\max}^2$$

If the mistakes in measurements are not taken into account ($\sigma_{nv}^{(i)} = \sigma_{ns}^{(i)} = 0$, $i = 1, \ldots, \mu$) and if the initial conditions are assumed to be realized accurately ($\sigma_{0v} = \sigma_{0s} = 0$), then for the optimal distribution of the correction instants we obtain the geometric progression formula

$$(\xi_i)_{\text{opt}} = \left(\frac{1-\theta_{i-1}}{1-\theta_i}\right)_{\text{opt}} = (1-\theta_\mu)^{-1/\mu} \qquad (i = 1, \ldots, \mu) \qquad (5.10)$$

This concentration of the correction instants toward the end of the trajectory is completely natural, since at the beginning of the motion, when there is still much time to eliminate the deviations from the computed trajectory, we can allow a larger amount of these deviations for a sufficiently small level of corrections Δa, i.e. we can let more time go by without corrections than at the end of the motion. The same distribution law for the correction instants was obtained in [4] where an impulse-corrected ballistic trajectory was investigated. However, in contrast to impulse correction (for which the errors are introduced only at the instants of application of the impulses), there does not exist an optimal number μ_{opt} of correction instants for the case considered in the present paper, where the errors are accumulated continuously (independently of whether corrections are made or not) under the condition of exact measurements. For given μ the minimum of the value of the mathematical expectation of the increase in the functional Φ

$$M(\Delta\Phi)_{\min} = \frac{6\varkappa L^2}{gT^3}\sigma_a^2\left(\frac{1}{3}\tau\varphi + \frac{1}{12}\right)$$
$$\left(\varphi = \mu\frac{2}{(1-\theta_\mu)^{3/\mu}} - \frac{3}{(1-\theta_\mu)^{2/\mu}} + \frac{2}{(1-\theta_\mu)^{1/\mu}} - 1\right) \qquad (5.11)$$

decreases monotonically with increasing μ, and moreover

$$\lim \varphi = -\ln(1-\theta_\mu) \quad \text{for} \quad \mu \to \infty$$

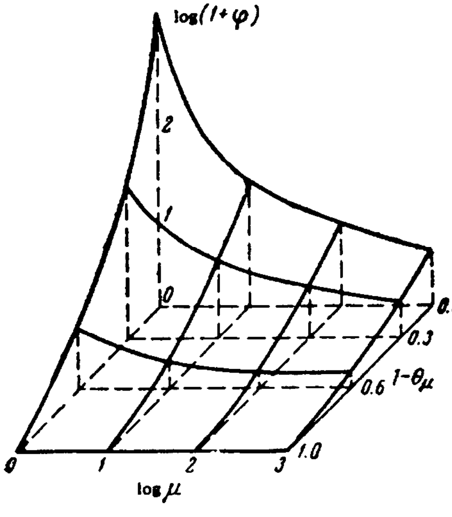
(see $\log(1+\varphi)$ as a function of μ and $1 - \theta_\mu$ in the figure). From the figure it is also seen that φ decreases with increasing $1 - \theta_\mu$, and therefore, the optimal value $(1 - \theta_\mu)_{opt}$ should be chosen from inequality (5.9) to be the maximum possible

$$(1-\theta_\mu)_{\rm opt} = \min\left\{\frac{1}{2}\frac{\delta v_{\max}^2}{\sigma_a^2\tau},\ \left(\frac{3}{2}\frac{\delta s_{\max}^2}{\sigma_a^2\tau}\right)^{1/3}\right\} \tag{5.12}$$

In the case of a nonzero constant dispersion of the measurement errors and deviations from the initial conditions ($\sigma_{nv}^{(i)} = \sigma_v$, $\sigma_{ns}^{(i)} = \sigma_s$, $i = 0, 1, \ldots, \mu$), the optimal distribution of the correction instants cannot be expressed by a simple formula, such as (5.10), and should be determined numerically. However, if formula (5.10) is used, then

$$M(\Delta\Phi) = M(\Delta\Phi)_{\min} + \frac{6\alpha L^2}{gT^2}\left\{\sigma_v^2\left[\frac{\theta_\mu}{(1-\theta_\mu)^{1+2/\mu}} + \frac{2}{3(1-\theta_\mu)}\right] + 2\frac{\sigma_s^2}{(1-\theta_\mu)^3}\right\} \tag{5.13}$$

The nature of the dependence of $M(\Delta\Phi)$ on μ and on $1 - \theta_\mu$, is retained.



However, if the dispersion of the measurement errors depends on the intervals between the correction instants, then, from formula (5.8), we can conclude that there should exist an optimal number μ_{opt} of correction instants for which $M(\Delta\Phi)$ has a minimum.

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Translated by N.H.C.